

## A Level Further Mathematics A Y545 Additional Pure Mathematics Sample Question Paper

Version 2

### Date – Morning/Afternoon

Time allowed: 1 hour 30 minutes

#### You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

#### You may use:

- a scientific or graphical calculator



### INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

## 2

Answer **all** the questions.

- 1 A curve is given by  $x = t^2 - 2 \ln t$ ,  $y = 4t$  for  $t > 0$ . When the arc of the curve between the points where  $t = 1$  and  $t = 4$  is rotated through  $2\pi$  radians about the  $x$ -axis, a surface of revolution is formed with surface area  $A$ .  
Given that  $A = k\pi$ , where  $k$  is an integer,
- write down an integral which gives  $A$  and
  - find the value of  $k$ . [4]
- 2 Find the volume of tetrahedron OABC, where O is the origin, A = (2, 3, 1), B = (-4, 2, 5) and C = (1, 4, 4). [3]
- 3 Given  $z = x \sin y + y \cos x$ , show that  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + z = 0$ . [5]
- 4 (i) Solve the recurrence relation  $u_{n+2} = 4u_{n+1} - 4u_n$  for  $n \geq 0$ , given that  $u_0 = 1$  and  $u_1 = 1$ . [4]  
(ii) Show that each term of the sequence  $\{u_n\}$  is an integer. [2]
- 5 **In this question you must show detailed reasoning.**  
It is given that  $I_n = \int_0^\pi \sin^n \theta d\theta$  for  $n \geq 0$ .
- (i) Prove that  $I_n = \frac{n-1}{n} I_{n-2}$  for  $n \geq 2$ . [5]
- (ii) (a) Evaluate  $I_1$ . [2]  
(b) Use the reduction formula to determine the exact value of  $\int_0^\pi \cos^2 \theta \sin^5 \theta d\theta$ . [2]
- 6 A surface  $S$  has equation  $z = f(x, y)$ , where  $f(x, y) = 2x^2 - y^2 + 3xy + 17y$ . It is given that  $S$  has a single stationary point,  $P$ .
- (i) (a) Determine the coordinates of  $P$ . [5]  
(b) Determine the nature of  $P$ . [3]
- (ii) Find the equation of the tangent plane to  $S$  at the point  $Q(1, 2, 38)$ . [2]

- 7 In order to rescue them from extinction, a particular species of ground-nesting birds is introduced into a nature reserve. The number of breeding pairs of these birds in the nature reserve,  $t$  years after their introduction, is an integer denoted by  $N_t$ . The initial number of breeding pairs is given by  $N_0$ .

An initial discrete population model is proposed for  $N_t$ .

$$\text{Model I: } N_{t+1} = \frac{6}{5} N_t \left(1 - \frac{1}{900} N_t\right)$$

- (i) (a) For Model I, show that the steady state values of the number of breeding pairs are 0 and 150. [3]
- (b) Show that  $N_{t+1} - N_t < 150 - N_t$  when  $N_t$  lies between 0 and 150. [3]
- (c) Hence find the long-term behaviour of the number of breeding pairs of this species of birds in the nature reserve predicted by Model I when  $N_0 \in (0, 150)$ . [2]

An alternative discrete population model is proposed for  $N_t$ .

$$\text{Model II: } N_{t+1} = \text{INT}\left(\frac{6}{5} N_t \left(1 - \frac{1}{900} N_t\right)\right)$$

- (ii) (a) Given that  $N_0 = 8$ , find the value of  $N_4$  for each of the two models. [2]
- (b) Which of the two models gives values for  $N_t$  with the more appropriate level of precision? [1]

- 8 The set  $X$  consists of all  $2 \times 2$  matrices of the form  $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$ , where  $x$  and  $y$  are real numbers which are not **both** zero.
- (i) (a) The matrices  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$  and  $\begin{pmatrix} c & -d \\ d & c \end{pmatrix}$  are both elements of  $X$ .  
 Show that  $\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} c & -d \\ d & c \end{pmatrix} = \begin{pmatrix} p & -q \\ q & p \end{pmatrix}$  for some real numbers  $p$  and  $q$  to be found in terms of  $a, b, c$  and  $d$ . [2]
- (b) Prove by contradiction that  $p$  and  $q$  are not **both** zero. [5]
- (ii) Prove that  $X$ , under matrix multiplication, forms a group  $G$ .  
 [You may use the result that matrix multiplication is associative.] [4]
- (iii) Determine a subgroup of  $G$  of order 17. [2]
- 9 (i) (a) Prove that  $p \equiv \pm 1 \pmod{6}$  for all primes  $p > 3$ . [2]
- (b) Hence or otherwise prove that  $p^2 - 1 \equiv 0 \pmod{24}$  for all primes  $p > 3$ . [3]
- (ii) Given that  $p$  is an odd prime, determine the residue of  $2^{p^2-1}$  modulo  $p$ . [4]
- (iii) Let  $p$  and  $q$  be distinct primes greater than 3. Prove that  $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$ . [5]

### END OF QUESTION PAPER

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